ABSTRACT. The article investigates in the first part critically dyadic and essentialist understanding of signs and utterances in mathematics and mathematics education as opposed to a triadic view. However, even Peircean semiotics, giving priority to triadic, dynamic sign may face challenges, such as explaining the sign as a pragmatic act and how signs are related to context. To meet these and other hurdles an explicit communicational, pragmatic and triadic view, found in parts of the works of Bühler, Bakhtin, Habermas, and Halliday, is developed. Two basic principles are combined and established in a theoretical framework. Firstly, whenever uttering, there will exist in any semiotic sign system, dynamic reciprocity and simultaneity between expressing through form, referring to content, and addressing as an act. Secondly, meaning will be created by the dynamics between given and new in utterances and between utterances and contextual genres. The latter principle explains how meaning merge in communication dynamically and create the basis for a discursive understanding of semiosis and hence even learning at large. The second part exemplifies each of the three main aspects and the dynamics of utterance and genre and given and new by excerpts from a textbook in mathematics education. The concept ‘positioning’, in use for operationalisation, is explained in relation to main principles of the framework. The article ends focusing crucial implications for validation when moving from a dyadic to a triadic understanding of mathematics and mathematics education.

KEY WORDS: communication, language, positioning, semiotics, theme-rheme, triadic utterance

1. FROM LOGIC MONADS VIA STRUCTURAL DYADS TO SEMIOTIC TRIADS

Mathematic education begins and proceeds in language, it advances and stumbles because of language, and its outcomes are often assessed in language (Durkin and Shire, 1991, p. 3). (…) But in an advanced culture much of that which has to be learned is stored in symbols. (Dewey, 1916, p. 8)

According to Maddy, mathematics as academic discipline, in its most purist form, tends to be seen as monadic, that is, it deals with singletons or closed numerable entities and categories that can be handled by mono-logic: “(…) every physical thing is already mathematical, and every mathematical thing is based on the physical. In place of the customary dualism of mathematical and physical, this pared-down set theoretic realism offers a version of monism.” (Maddy, 1990, p. 157)
Following a monadic view mathematics as a discipline tends to be treated as a ‘content box’ of single elements where, for instance, mathematical signs such as $1$, $+$, $=\!$, $0$, $-$, $\div$, $\times$, $\pi$ etc tend to be seen as purely logical phenomena that should be focused as mathematical objects. An educative implication of such a category-based view is often that learning mathematics basically is about understanding singular logical elements. Pedagogically mathematical objects accordingly need to be presented to learners in a certain logical order relative to the estimated complexity of each mathematical sign. Hence ‘objects’ such as $1$ and $+$ are learned before $0$ and $\pi$ and are taught according to a ‘categorical’ view.

A somewhat different position is taken by so-called logical positivism or logical empiricism, represented by for instance Carnap and the early Quine. According to Hersh mathematics for this philosophical position is nothing but a language for science, a contentless formal structure (Hersh, 1997, p. 167, my italics). Hersh refutes their positioning by claiming that the positivist view of mathematics as a tool for science obstructs seeing mathematics as a mental activity in itself. Hence a sign based, positivist understanding could phenomenologically and logically be seen as dyadic, perceiving mathematical elements as binary signs based on a Saussurean-like model of semiology, segregating between the signifier (the form or the structure of the sign) and the signified (the semiotic ‘representation’ or content the signifier signifies).

Mathematics, although useful and even necessary for science can even be seen as a language in its own terms, not only as a language for science. According to what one could call a neo-Platonist understanding, mathematics can be perceived of as a full sign system, as a ‘language’ in its own right or even as a set of interrelated, semiotically different languages or sign systems. Nevertheless may a dyadic understanding still tend towards objectivism by prioritising the mathematical objects, because there exists, at least in Euclidian mathematics, a fundamental need for ‘closing’ each ‘object’ to avoid uncertainty. Certainty is achieved mainly by the use of axioms and interrelated systems of definitions, which is at the very heart of mathematics as an academic discipline and logic enterprise.

However, if we turn to the system of possible mathematical signs, or the system as meaning potential in Halliday’s term, things become more complex. Semiotic systems, as for instance verbal languages, can in principle, depending on the theoretical point of departure, either be seen as systematically closed or as systemically open. In the following I will clarify the implication of a systemic view. A system can be seen as closed when it is unchangeable, but systemic when it balances permanence with changes. Thus if one focuses a system in a Saussurean, synchronic sense (la langue), it is in principle seen as closed. Similarly mainstream mathematics then,
will extract the relative stable, logic elements and notions from ‘natural language’ such as for instance ‘and’, ‘larger than’ or ‘parallel’, and interrelate and purify them to achieve a functional or perfect predictable closure. Hence according to this view mathematics consists of given and added content elements similar to a (growing) vocabulary, and some stable, given grammatical, structural, syntactic rules.

Nevertheless should even this paradigmatic view be seen as dyadic, since it is based on a digital, binary logic, that is, an either-or thinking that helps to fixate meaning and to establish systems based on a categorical and dyadic view. The grammar will therefore basically consist of semantics (content elements) and syntax (set of acceptable rules for meaning structures), that is, a dyadic conceptualisation. Such a system is seen as a whole, and then used. Not in any sense is this system seen as disturbed by what it does or how it functions. Neither the use-aspect nor the context is seen as part of the system. Saussure left out the use aspect of language, what he conceptualised as ‘la parole’, arguing that application was too indefinite to be incorporated in language as a system, ‘la langue’.

However to see meaning as action in the functional tradition of Bühler, Wittgenstein, Austin and Searle, and consider language as a tool or a means, may lead to a dynamic, systemic and hence a triadic understanding. This view will take on the character of communication, not just ‘language’, as it will need to incorporate, along with and relative to semantics and syntax, a third aspect or logic, namely pragmatics. The utterance $2 + 2 = \_\_\_\_\_$ is hence not just four semantic elements, categories or objects in a syntactic structure to be read from left to right, but even a dynamic pragmatic act, with several implicit functional elements. For instance, it could pragmatically be read as Take two, Add two and Tell us what one Gets. In any case this utterance and any other utterance could be interpreted as a recognised, context-bound genre, since we apply genres to make sense of utterances.

Accordingly, a paradigmatic philosophical, logical and functional conflict may arise when we relate, reciprocally, syntax, semantics, and pragmatics; language can be seen just as a ‘tool’ for communication, whereby semantics and syntax work as dyads closely together and after when pragmatics is added in a kind of two-step process, so to speak, separating ‘la langue’, (the system), and ‘la parole’, (the use). Or one can see syntax, semantics and pragmatics as closely intertwined, balanced, reciprocal and inseparable, as one. In semiotics Charles Morris probably was the first to point to the interrelatedness of these three constituting aspects of signs (Morris, 1938 and 1946). However Morris’ triadic view has been controversial, especially among interpreters of the work of Peirce (Nöth, 1990).

This dilemma between parts and a whole is relevant not only for verbal language, but for any ‘semiotics’ or meaning based sign systems. In
this situation a deliberate move away from a dyadic and static Saussurean semiology to triadic dynamic Peircean semiotics may therefore at a first glance help seeing sign, not as static, but dynamic. The Peircean ‘third element’, the interpretant will function as the crucial element in explaining the (indefinite) process of change in the meaning of the sign, the semiosis. (See different approaches, this volume.) However this position, seeing the sign as mainly mental, rather than as an act, related to and dependent on context, is problematic, even if it takes into consideration a dynamic view of sign, text and discourse.

2. “TEXTS AND THE OBJECTS OF MATHEMATICS” OR PEIRCE OR HABERMAS?

I as eye make words of worlds, acts of facts and signs of science or vice versa?
(Ongstad, 1997, p. 434)

In Ernest (1998) it is stated that (…) the social constructivist view is that the discourse of mathematics creates a cultural domain within which the objects of mathematics are constituted by mathematical signs in use. (Ernest, 1998, p. 193.) There are good reasons for agreeing that the relationship between the particular sign and the general discourse is related to their function (use), even in mathematics. Ernest has two (inter-)related concerns: How can a positive aspect of Platonism, (the ability of explaining the nature of mathematical objects), be kept without being captured by its (negative) ontology? And: What is the relationship between the objects of mathematics and texts?

While Ernest (1998) on the one hand has advocated for a Peircean approach to mathematics, Jürgen Habermas on the other hand has pointed to, in his view, a weak spot in Peirce’s semiotics. Habermas raises a basic question:

What considerations could have induced Peirce to turn away from the intersubjective aspects of the sign process? I want to defend the thesis that the interpretant relation of the sign cannot be explained without recourse to the conditions for reaching an intersubjective agreement, however rudimentary these conditions may be. (Habermas, 1998/1988, p. 92)

In concluding, Habermas holds that the legacy of Platonism is repeated in Peirce’s work, in spite of the fact that Peirce seems to take an anti-Platonist standpoint:

Both allow communication, in which tendency to universalisation asserts itself, to be seen from only one side: communication is not for the sake of reaching
mutual understanding between ego and alter about something in the world; rather, interpretation only exists for the sake of representation and the ever more comprehensive representation of reality. This privileging of the sign’s representative relation to the world above the sign’s communicative relation to the interpreter causes the full-fledged interpreter to disappear behind the depersonalized interpretant. (Habermas, 1998/1988:109)

Habermas points to the fact that the young Peirce actually worked with the idea of writing a book about the interplay and the dynamics of ‘I’, ‘Thou’, and ‘It’ as “Elements of thought” (intended book title). However he did not stick to the idea. It seems, according to Habermas, that this is the closest Peirce came to a communicative/intersubjective understanding of semiotics, irrespective the fact that Peirce meant that “all thoughts were addressed to a second person” (the so-called doctrine of Tuisim) (Habermas, 1998/1988, p. 111, note 3).

Accordingly, if Ernest in his thorough and quite convincing critique of absolutisms (Ernest, 1998) at the end of the day nevertheless lands on a Peircean semiotics as a social constructivist base for understanding mathematics (Ernest, 1997, Ernest, this volume), he may end up like Peirce, stopping Platonism at the front door, but letting it in by the back-door. Thus I agree with Habermas that the intersubjective aspects (‘society’, ‘you’, ‘act’) should be brought into a real triadic framework, not a dyadic. The framework I will present though does not allow for a simple, dyadic either/or logic, but will rather consider the problem as a general logical conflict between both/and, that is, between dyads and triads, or in other words, between a logic of language as dyadic and a logic of communication as triadic (Morris, 1938, 1946). This is not to deny that logical ‘instruments’ such as the dyadic sign and binary logics are necessary for creating researchable objects in different sciences, included formal mathematics. But even ‘pure’ mathematics can be perceived from the perspective of pragmatics, by seeing mathematical utterances as a dynamics of syntax, semantics and pragmatics.

However, if we turn to the field of mathematics education (the Didaktik of mathematics), as different and separate from mathematics as an academic discipline, this field could perhaps not only benefit from, but probably even need a triadic framework to escape antagonisms and unfruitful polarisations. In the cultural domain, not the least in educational disciplines, it is hard to find any mental, human activity that does not have a communicational aspect or could not be seen in a communicational perspective. Thus Dewey claims that: Not only is social life identical with communication, but all communication (and hence all genuine social life) is educative (Dewey, 1916, p. 5). Semiotics as a field has been defined as the study of culture as meaning, and a pragmatic or a functional, triadic, oriented semiotics could
therefore help to bring to surface precisely the deep and basic connection Dewey is pointing to.

Hence, it may help understanding how mathematical concepts (that is, logical signifiers) are developed diachronically and historically, how growth in insight in a given phenomenon can be a cultural object for teaching and learning. Such a perception may nevertheless still end up being sign fixated and not functionally oriented. An object oriented view often loses sight of the dynamic nature of an authentic utterance (as different from an object-like, theoretical grammatical sentence). An utterance cannot be seen as an object, cut off from its context without ‘killing’ it as an authentic utterance (Bateson, 1972; Bakhtin, 1986; Ongstad, forthcoming a). It should be handled both as a true triadic phenomenon, that is, as inseparable dynamics of syntax, semantics and pragmatics, and as a dialogical phenomenon, constantly balancing and blurring the given and new when uttering, saying new things with given means (Bakhtin, 1986). Thus in the following I will present two basic, interrelated principles, a triadic and a dialogical view of utterances and genres, and then combine them in an overall framework. The basic idea is to help understanding mathematical utterances in educational contexts, not just as syntax and semantics, and with pragmatics added to them, but as triadic communicative, semiotic utterances, as integrated syntax, semantics and pragmatics. In this sense both different kinds of mathematics and the whole field of mathematics education can be seen as triadic, semiotic communication.

3. INDEPENDENT ASPECTS OR INTERRELATED TRIADS?

Society exists through a process of transmission quite as much as biological life. This transmission occurs by means of communication of habits of doing, thinking and feeling from the older to the younger. (Dewey, 1916, p. 3; my italics)

Through the history of rhetoric, philosophy, communication theory, didaktik and education there are developed and transmitted quite a few sets of tripartite, overarching aspects or so-called triads such as pathos-logos-ethos, aesthetical-theoretical-practical (Kant, 1987), beauty-truth-goodness, heart-head-hand (Pestalozzi), I-it-you, form-content-use, and teacher-discipline-learner. An unpretentious, but symptomatic example of such general thinking in three can precisely be found in the above quote: feeling, thinking, and doing. Recently other theorists and educationalist have coined, used or composed similar triads: personal being-physical being-social being (Harré, 1980, 1984,
experience-understanding-action (Hiim and Hippe, 1998), psychic-cognitive-social, respectively Freud-Piaget-Marx (Illeris, 1999), child-object-mother (Poulsen, 1994; Hansen, 2000), the aesthetic-the cognitive-the moral (Bauman, 1995), student-subject-teacher (Künzli, 1998; Westbury, 1998), aesthetic-scientific-moral (Miller, 1998) and beauty-truth-goodness (Aas, 2003). It should be underlined that the three aspects in all the above sets should be seen as mutually interrelated. As real triadic aspects they define each other reciprocally. The problem though is that what actually connects them hardly ever is made explicit.

However more explicit explanations are found in the work of Bordum (2001), Bühler (1934), Habermas (1984, 1988, 1998/1988), Halliday (1978, 1994), Hansen (2000), Hernadi (1995), Martin (1997), Ongstad (1996, 1997, 1999a,b, 2002a,b, 2004a). If we start from a general level Habermas holds that our total communicative lifeworld consists of three interrelated ‘worlds’, the objective worlds, about which there can be made true statements, to the social worlds which consist of all interpersonal relations and to the subjective worlds that are all experiences to which only the utterer has privileged access (Habermas, 1984, p. 100). The processes, the aspects and the relations will inevitably be triadic:

Thus, to the different structural components of the lifeworld (culture, society, personality) there correspond reproduction processes (cultural reproduction, social integration, socialization) based on different aspects of communicative action (understanding, coordination, sociation), which are rooted in the structural components of speech acts (propositional, illocutionary, expressive). These structural correspondences permit communicative action to perform its different functions and serve as a suitable medium for the symbolic reproduction of the lifeworld. (Habermas, 1984:xxv, translator’s introduction)

Based on ideas from text theory and semiotic genre theory, Habermas’ more general ideas can be narrowed down and made more concrete, leaning to Bakhtinian triadic and dialogical notions of utterance and genre. According to Bakhtin (1986) an utterance is a delimitable unit of communicational meaning which, when (partly) repeated, enters into a dialogical relationship with genres (types of utterances or kinds of communication). To utter consists of three different phases, opening, positioning of aspects and sufficient finalisation. (Those three sequenced elements should not be confused with the kind of reciprocal triads I have pointed to above.)

When positioning and finalising, the utterer combines the use of more or less stable generic forms (style or syntax), semantic references and a strategic, pragmatic speech plan (Bakhtin, 1986, pp. 76–99 Ongstad, 2004b).

All of these three triadic aspects, respectively expressivity, referentiality, and addressivity, occur in any utterance and will be taken from available
meaning potential (genres) to utter in communicatively different ways. The utterers have to position actively in relation to the three mutual processes of expressing, referring, and addressing. Thus any utterance will inevitably have three reciprocally defined dimensions, aspects and processes which will link the utterance respectively to the utterer, to the world and to others, in other words to ‘self’, ‘world’, and ‘society’. These three overarching aspects should correspond respectively to Habermas’ terms person, culture and society.

There are striking affinities and similarities between the above aspects of Bakhtin’s work and a famous, classic communicational triad, Bühler’s so-called organon model (Bühler, 1934). The use of the Greek word ‘organon’ hints that language is seen as a means, not just as a system or as dyadic opposition between a signifier and a signified, as we find in Saussure’s influential work (Saussure, 1916). Bühler’s paradigmatic new pattern puts him in some sense in line with later scholars such as Wittgenstein, Austin, Searle and Halliday, all stressing language, not in, but as use, in other words basically a functional or a pragmatic view.

According to Bühler there are three main characteristic functions of language and of signs in general: expression, representation, and appeal respectively related to sender, objects, and receiver. However, the very act of focusing gives each function a (possible twisted) dominance. Hence the sign is a symptom when expression dominates (or is given communicative priority), a symbol when the representative function dominates and a signal when appeal dominates. Nevertheless will all aspects be at work at the same time in any communication (Bühler, 1934, p. 28; Nöth, 1990, p. 185). Bühler is historically the direct source and inspiration for Habermas. In Toward a Critique of the Theory of Meaning, Habermas explicitly underlines that a validity-theoretical interpretation of Bühler’s functional scheme (. . .) leads to the assumption that with a speech act ‘MP’, S takes up relations

![Diagram of Bühler’s organon model](https://example.com/diagram.png)

*Figure 1.* A simplified version of Bühler’s organon model of language. For a fuller version, see Nöth (1990, p. 186) and Bühler (1934, p. 28). (S = sign.)
simultaneously to something in the objective world, to something in the subjective world, and to something shared in a social world. (Habermas, 1998/1988, p. 76, Habermas’ own underlining.)

To accept a triadic simultaneity though will challenge traditional perceptions of validity, which often build on a dyadic/digital/dual thinking. The potential openness of simultaneity in communication may of course be refuted by (pure) mathematics, as a discipline searching and building on accuracy, but will nevertheless need to be brought in again in mathematics education, since any Didaktik is communicational. The triadic communicational perspective implies a performing mind, not only a description of abstract symbolic representation of an objective world of phenomena cut off from users and contexts. This principle of communicational wholeness of expressivity, referentiality and addressivity implies that validation of any utterance in a broad sense should be seen as reciprocal dynamics of subjectivity, objectivity and normativity. Mostly there will be no clear lines between these aspects in real utterances, although the genre or kind of communication may hint what is intended to be dominant.

‘Translated’ into more traditional terms Bühler’s triad equals or parallels a traditional triad of form, content, and use in utterances and texts. These notions are now frequently used in textbooks for mother tongue education in primary and teacher education in different countries. Hence a sign, an utterance, a text, a genre, a discourse, a context, in short, any communication will have a structural (form), referential (content), and an addressive (use) aspect. These are always simultaneously intertwined, which in practice means that meaning and real communication happens within and because of the dynamics in this triangle. Or in other words, the triadic mutuality functions as the very definition of communication.

A common hesitation when faced with the triadic claim is why it has to be just three. The basic aspects are three, and not two, four, five, six or seven, because three is the lowest possible number of basic aspects necessary to establish or define communication (as different from ‘language’): Someone expresses something to someone. The three should not be seen as segregated categories, but rather as dynamic aspects or functional ‘vectors’ as part of a certain wholeness. They create the final utter-walls of a semiotic ‘universe’ or ‘lifeworld’ within which utterers must communicate. Each of these aspects constantly risks to break down though. Where there is no structure or no reference or no addressivity there is also no communication.

Thus an overview over some of the triads we have met so far could simplistically be structured and interrelated the following way:
The reader is asked not to focus too much on each aspect and to get the overview ‘explained’ at this stage. What I claim so far about all these aspects and patterns is just that there is a systemic relationship between all of them. They have something to do with each other that is not a coincidence. Thus there is a horizontal connection between aspects of singular triads as well as a vertical between aspects connected to each main dimension of communication, or in other words within and between these (and many other) sets of communicational triads. In these two respects they all form a systemic, communicational system. Another main claim is therefore that for any communicator, not the least in research and education, this system is inevitable: one has to enter the framework from certain positions dependent on the disciplinary interest, and at the same time having to position relative to all aspects.

However there are pitfalls connected to this understanding that need to be avoided. One is connected to the logical, paradoxical conflict between a categorical and a relational thinking as pointed to in the introduction. A main logical mistake when presenting such triads is precisely to give readers the impression that they are separate categories which can be easily placed in separate rows and columns (as done above). This visual simplification is partly Habermas’ problem, even if he explicitly has stated that each of his triads forms a whole.

However, by grouping the triads explicitly as mutual relations, one will get a systemic understanding that will alter the logic. By systemic is meant the dynamic and partly unpredictable character of a relational system as different from a closed, systematic stable system. (I earlier pointed to openness as one of the characteristics of systemic semiotics.) The mutuality implies to take the full consequence of the principle of simultaneity between triadic aspects. One of the possible critiques could be that interrelating them

<table>
<thead>
<tr>
<th>Main aspect of utterance</th>
<th>Habermas’ ‘worlds’</th>
<th>Didaktic dimensions</th>
<th>Sign aspect (Bühler)</th>
<th>Functions (Bühler)</th>
<th>Main aspect as function</th>
<th>Part of language study</th>
<th>Utterer’s main activity</th>
<th>Field of human activity</th>
<th>Aspects of utterance</th>
<th>Validity connected to</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>form</td>
<td>content</td>
<td>use</td>
<td>principal experience</td>
<td>teacher</td>
<td>symptom</td>
<td>expression</td>
<td>psychic</td>
<td>expressivity</td>
<td>subjectivity</td>
</tr>
<tr>
<td>Habermas’ ‘worlds’</td>
<td>the subjective worlds</td>
<td>the objective worlds</td>
<td>the social worlds</td>
<td>personal experiences</td>
<td>mathematics</td>
<td>symbol</td>
<td>representation</td>
<td>cognitive</td>
<td>referentiality</td>
<td>objectivity</td>
</tr>
<tr>
<td>Didaktic dimensions</td>
<td></td>
<td></td>
<td></td>
<td>mathematics</td>
<td>mathematics</td>
<td></td>
<td></td>
<td></td>
<td>referring</td>
<td>normativity</td>
</tr>
<tr>
<td>Sign aspect (Bühler)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>symbol</td>
<td></td>
<td></td>
<td></td>
<td>doing</td>
<td>normativity</td>
</tr>
<tr>
<td>Functions (Bühler)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>pragmatics</td>
<td>normativity</td>
</tr>
<tr>
<td>Main aspect as function</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>addressive</td>
<td>normativity</td>
</tr>
<tr>
<td>Part of language study</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>normativity</td>
</tr>
<tr>
<td>Utterer’s main activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>normativity</td>
</tr>
<tr>
<td>Field of human activity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>normativity</td>
</tr>
<tr>
<td>Aspects of utterance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>normativity</td>
</tr>
<tr>
<td>Validity connected to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>normativity</td>
</tr>
</tbody>
</table>
can lead to circular definitions of triads, which mainly would be the case if they were seen as categories. However, the circle argument is based in a dyadic paradigm and is not compatible with a wish to understand communicational utterances as complex meanings. A triadic view implies an interpretative or a hermeneutic approach. The framework nevertheless allows for an essentialist aspect to be part of this wholeness, what Habermas would relate to the objectivity part of the validation, and what one could see as validation of the referential aspects of utterances, evaluating them as true or false. In other words, should all communicative aspects be interrelated in a systemic system.

‘Systemic’ here is close to the logic of the notification system in chemistry, allowing for changes and transgressions between different elements. For more thorough theoretical discussions of triads, see Habermas (1984), Hernadi (1995), Bordum (2001), and Ongstad (1999a, 1999b; 2002a,b, 2004a,b). Communicational triads will form a systemic open universe in-between which communication proceeds, and to which communicators have to position themselves, the topic and the receiver.

4. DYNAMIC TRIADS AND THEME-RHEME PROCESSES IN UTTERANCES – A FRAMEWORK

Society not only continues to exist by transmission, by communication, but it may fairly be said to exist in transmission, in communication. (Dewey, 1916, p. 4)

So far I have outlined the first of the two principles I announced would be a main part of the framework. In the following I will develop the other part. In The Politics of Mathematical Education, Stieg Mellin-Olsen discusses differences between Saussure (1916) and Ogden and Richards (1923) to conceptualize meaning (Mellin-Olsen, 1987). He is dissatisfied with both views since they lack, not only a connection to activity, but even a theory of context, which seems to be necessary to get an appropriate understanding of meaning, a conviction he got from Bateson (1972) and others. In Halliday
Figure 3. The principle relationship between the three major aspects on the concrete level of utterance/text (the top triangle) and their respectively corresponding three ‘lifeworld’ aspects of the immanent level of context/genre, (the ‘bottom part). The white arrows point to the two levels in the figure, the top surface and the rest.

(1978) he finds a more contextual, functional and pragmatic approach to communication. Even if he pays attention to Halliday, Mellin-Olsen nevertheless seems to overlook that Halliday not only advocates for a contextual grammar, but offers an approach that combines both textual triads of syntax, semantics and pragmatics and dynamics of text and context.

The Hallidayan double combination represents a rather advanced socio-semiotic understanding, and is primarily developed as a systemic-functional textual grammar (Halliday, 1994; Martin, 1997; Morgan, this volume). However, in the following I will present a somewhat similar and perhaps even a compatible framework, based on other sources. My extended, more general framework aims at including all semiotics, not only verbal or textual communication, which in my view is an important grip to avoid treating mathematics and mathematics education single-mindedly from the perspective of verbal texts.

There are two main processes connected to the model, marked by the arrows A and B, a parallel blending of form, content and use of the aspects of the utterance, that from an analytical point of view can be studied as dynamics (positionings) of structure, reference and action (A) while uttering as I have already indicated. The other kind of process (B) happens between the text/utterance level and earlier stored elements in the utterer, or meaning potential in Halliday’s terms. These are taken from deeper levels of the mind and the body, what I have called the level of context/genre. The stored elements work as active resources in combining given and new elements to make meaning in particular communicative situations. These mechanisms, so-called theme-rheme processes are intricate dialogical relationships between utterance/text on the one hand and kinds of communicational situations or genre/context on the other hand (Halliday, 1994).
To simplify, in any utterance, not the least in teaching and learning of mathematics, there is a discursive blurring of given parts (theme) and new parts (rheme). The balancing and shifts of these elements happen as the text (the utterance) unfolds for the utterer as well for the interpreter. Different semiotic systems and different communicational cultures (and genres) have different syntactic-semantic-pragmatic expectations for these dynamics.

Further the text/utterance will always be concrete (metaphorised as the triadic surface in the figure) and the context/genre will be immanent, that is real, but not ‘visible’ (metaphorised as the rest under the top surface). Accordingly there are tight logical connections, respectively and as a whole, between structure and ‘self’, reference and ‘world’ and action and ‘society’. Just to exemplify very briefly, for instance, if our physical national/regional/subjective language form becomes part of our ‘selves’, our input of references to mathematical phenomena will build up some kind of personal, disciplinary world-view of mathematics, and our communicational actions in the family will relate us to or constitute a major part of our embodied ‘society’. Again, there are no ‘walls’ between these relationships/processes (the cut pyramid is undivided), which means that a ‘self’ is developed even in relation to reference and actions; for an inner ‘world’ to develop it is dependent on action and structure as well; and for an embodied society to evolve in a person, not only action, but reference and structure is needed too.

Utterances are on the one hand partly being created by the use of the already embodied resources or meaning potential in Halliday’s term. On the other hand they turn into stored potential for future production and interpretation of utterances. What helps in keeping this enormous and rapidly growing storage in order, is genre. Or to phrase it differently, human beings have an amazing capacity rapidly to use, recognise and differentiate between kinds of communication. In other words, a differentiated genre system will function as an advanced, constantly accumulating meaning potential for all kinds of communication (Ongstad, forthcoming b). It offers certain communicationally tacitly agreed pre-balances of form, content and use that most sign users can recognize. For example, formulas will give priority to structure/form, definitions to reference/content, and instructions/tasks to action/use. These three genres can serve as three prototypical border examples, even if utterances in these genres simultaneously of course express, refer, and address. However, most of the perhaps thousands of other culturally ‘negotiated’ genres or ways of communication will be found in the huge discursive landscape in between. To draw a sufficiently distinct line between them is normally a necessary and a risky business, both for communication, interpretation and validation. However for research and education this search is inevitable.
5. Exemplifying the Three Constitutive Aspects of Uttering

5.1. Example A

A, E, F...
B, C, D...

Before you read further, spend some time considering what you think will be the next letter in each of the two rows above. The answer is further down in the text. In the following I will mark with italics the concepts related to the focused aspect that is systemically part of the framework. The most immediate and direct aspect of utterances is their appearance, their physical, substantial form, depending on the medium or the channel through which they are brought to us. Mathematicians may for instance have an inclination to try to solve what appears to be a logical problem, for instance of the above kind. They are enculturated to see such utterances as challenges or tasks which can be solved based on more advanced knowledge of arithmetic sequences. However, young children, learning the alphabet in mother tongue education, may in the case above see the ‘answer’ before the mathematician: H in row 1 and G in row 2 (rectilinear versus ‘roundish’ letters).

Sáenz-Ludlow (2003) has a similar example where a mother asks a child: What is half of 8? and the child answers 3, focusing on the right half part of 8 configured as a structural shape. Hence when form is used for communicational purposes, it will be given a structure. This structure can be studied as or from the perspective of syntax. Syntax should not be understood in a restricted linguistic sense though. The form has direct impact on and connections to our feelings/ emotions. When we utter, especially orally, our body is part of or actually is the form of the utterance (our dialect, our personal style, our aesthetic appearance and so forth). Because we are personally part of our structure when we utter, we have to be interpreted as something subjective, as a Bühlerian symptom.

5.2. Example B

T: What is the difference between 8 and 3?
S: Well... they are different. 8 is bigger than 3.
T: If you take away 3 from 8, how much is that?
S: That is 5.
T: If you subtract 3 from 8, how much is that?
S: That is also 5.
T: What is the difference between 8 and 3?
S: I told you before. 8 is bigger than 3
(From Sáenz-Ludlow, 2003.)
What we see demonstrated here is a *semantic* difference regarding the conception of the word ‘difference’ between the teacher who is enculturated to the *discipline or genres of mathematics* and to the student who is not (yet). There are several *references* to the form ‘difference’, the two operate with different *contents*, dependent of their different embodied contexts (conceptualised by different theorists as for instance register, genre, discourse, code, habitus). Once *subject matters are understood*, they become *knowledge*.

5.3. *Example C*

8 − 3 =

Teachers and students of mathematics are enculturated to handle the structure in example C as something to do, a task. More or less automatically quite many will *perform* the calculation because the *addressivity* is interpreted as an *invitation* or an *appeal*, a *signal to do subtraction* based on the interpretation of the signs as *operators*. If the enculturation is strong, this tends to happen irrespective of other contexts in which the utterance occurs: *We cannot not subtract*. The ideology of this mathematical genre/discourse thinks for us. Thus a *pragmatic* positioning (reading) of the utterance may reveal such ideologies.

I will underline that the exemplifications in the above utterances are my own conscious *positioning(s) of main communicational potentials* and that *each* of them of course can or should be seen as simultaneous mixtures of form, content and/or use. From the point of view of mathematics education form can be associated with emotion, content with cognition and use with activity and thus be related to, respectively, the aesthetic, epistemological and ethical field. The above, partly fabricated and deliberately ‘naive’ examples may seem simple, but the educational importance of for instance form, tends to be underestimated in mathematics education. Doing mathematics is not just a matter of combining logic and performance, it is a profound affective, emotional and hence aesthetic experience as well (Ongstad, 2004a).

Aesthetic here implicates positive, negative as well as ‘neutral’ aspects. Mathematics is hated and loved, it is horrible and beautiful, clear and obscure, negatively frustrating and positively challenging (Kragh, 1999). The aesthetic dimension is not general and static, but always shifting through the processes of uttering and interpreting. The communicator *evaluates* the parts of the utterances, consciously and unconsciously, through which the psyche is inevitably attached to the uttering process. In Bakhtin’s term utterers will utter with a certain *expressivity*. Expressivity should not be confused with ‘strong emotions’. Any utterance has a certain expressivity.
In Bühler’s term communicators interpret the utterance as symptom if they try to interpret from expressivity to utterer. Expressivity is everywhere in an utterance, but may be more explicit and outspoken in some parts than in others (in mathematical discourse sometimes in exclamation marks!). When we give expressivity priority, we tend to communicate emotionally.

In all cases we need to validate not only by focusing the researched ‘object’, but even by analysing the aspects we leave behind (what is not in the focus). Hence, to validate is a direct discursive and a parallel meta-communicational process. What we interpret is a dynamics of the focused ‘figure’, its immediate (back-)ground and the position from which we focus.

My last remark is that all the three examples are presented more or less as ‘categorical’, non-relational, and without context, a segregation I warned against in the introduction. Therefore, any serious or final interpretation would in this case be quite meaningless. The main idea has been to focus on each of the three aspects as such, when seen as separate.

In the following section I will exemplify how genres contribute to what can be seen as context, and focus on how given and new are blurred to give new meaning. I first present the basic dynamics between the two, and then, as an example, the dynamics in relation to one particular aspect, namely expressivity.

6. Theme-rheme or new and given in mathematical genres

If we imagine the utterance as an equation and decoding as a solving process, we have to find the unknown entity in very different places in the equation depending on what is known beforehand (…) The equation and the verbal mediated message can thus be compared both regarding complexity and progression. In the mathematical equation there might be one or more unknown elements, in the message one or more fragments of new information. In the equation the unknown will be made known by a systematic combining of all entities, in the decoding of the message the new information has to be framed by all that we know or believe we know (the free information). (Rommetveit, 1972, p. 137, my translation)

There are many different definitions of theme. Halliday (1978, 1994) sees theme as a function in the clause as a message: it is what the message is concerned with – the point of departure for what the speaker is going to say. A more general view, not only related to text and verbal language, is that theme elements can be related to genre, since a genre precisely presupposes much of what can be expected in the kind of communication in question. (Much is given.) The rheme-parts are the new parts, and have always to be combined with the given in order to function as real communication, that is to give (new) meaning. If all elements would have been theme (an already
total given utterance) or everything would have been rheme (a total new
utterance) there would be no (new) real communication. Thus the more
the theme is (strictly) given, the closer the utterance is to a (stereotyped)
genre, and the more the positioning is already pre-scribed by the genre
for the utterer. Examples could be proofs and definitions in mathematics.
Further, the more new elements there are, the more risky or challenging
positioning as communication may be for the utterer, as well as for the
receiver/interpreter. Examples could be ‘complicated’ poems and abstract
paintings. However the given and the new will shift syntactically, seman-
tically, and pragmatically through the whole process of uttering and will
normally be difficult to trace analytically in every detail. Halliday therefore
holds that each analyst should choose the level of ‘delicacy’ according to
the analytical function of the research in question (Halliday, 1994).

In the following example, seemingly a beginning of the genre ‘fairy
tale’, I have marked some possible themes and rhemes as they appear in
the text. One should have in mind that grammatical presentation construc-
tions mostly are ‘new’ and normally occur in indefinite form (rheme): A
man normally appears in a text before the man. Once mentioned however
the rheme element is now known and can in the next text step get the def-
inite article and function as a known theme. This kind of shift is also the
basic principle which theme-rheme dynamics will follow more generally.
Moreover, it represents and reflects the way we learn, adding new to given,
which again becomes a new, more advanced ‘given’ and so on. This is the
case not only for particular concepts, but for understanding and develop-
ing knowledge and skills at large. Hence, the theme-rheme process is the
discursive key to semiosis, to the never-ending change in meanings, or in
other words even to learning (Ongstad, 2003).

[Once upon a time = theme] [a linguist wanted to visit a mighty castle
called mathematics education = rheme]. [The castle = theme] [had huge
trolls guarding = rheme] [it = theme]. [But the linguist = theme] [knocked
boldly on a door called semiotics = rheme] etc.

My main idea here is just to make aware how crucial theme-rheme pro-
cesses actually are for the planning of progress and development in teaching
and learning. Particular genres such as textbooks, definitions, explanations,
and proofs for instance, will often have an implicit regime for balancing
theme-rheme (or given and new) that we learn to use and recognise.

However, sometimes rheme elements appear outside ‘the zone of ex-
pectation’:

On the 18th of April 1994 class 7X (age 14) is doing algebra. They are solving
textbooks tasks (calculating by insertions). There are quite a few seemingly similar
tasks. Suddenly there is an air of confusion and slight irritation. One of the tasks, on the surface similar to the others, seems ‘wrong’: Calculate $3x - 7y$ for $x = 3$ and $y = 2$. The frustration is eventually so strong that the teacher, Hedvig, has to stop the competition race and do the calculation in plenum on the blackboard (Ongstad, 1997, p. 363).

This is a so-called ‘insertion’ task. Seemingly it looks like many other small ‘sums’ (arithmetic tasks) the class has had the last days. “Calculate” represents a clear genre signal. It is not ‘find’ or ‘consider’. A calculation is expected. The operation required is the topic (the theme) of the day, insertion. The theme part is ‘calculate’. The rheme (the new) elements are any new logical units appearing linearly from the left to the right (in Western cultures). The sums have stereotypical patterns with little variation, which soon establishes safe genre patterns, since all tasks in the textbook look quite similar. However, as Sáenz-Ludlow (2003) points out, the novice learner has an asymmetric pre-knowledge compared to the advanced teacher. In other words the better one knows the genre and the genre system, the better the readiness (zone of expectation) a receiver may have for what might appear. This genuine genre knowledge helps, mathematically, to guess how this piece of calculation may unfold in the theme-rheme process.

For the novice learner the task above at a first glance seems trustworthy. It looks like many of the tasks 7X has had earlier that semester. Most of the students will therefore not suspect any complex rheme part to occur, since this ‘genre’ has established itself with easily recognizable patterns. The students will consider the task utterance as a combination of safe theme elements they already know, where the only new elements are the specific figures to be filled in. Their problem is therefore experienced rather dramatically. They have never met a negative number as answer in this genre. Most of the students therefore found this task ‘wrong’ or ‘impossible’. They gave up.

The teacher had to explain this carefully. The students had of course heard about ‘negative numbers’ earlier, but they could not imagine the possibility of a negative number in this context or genre. The original taskmaster (the textbook writer) of this particular utterance had placed this ‘rhematic’ element more or less consciously as a ‘surprise-bomb’, hidden within a seemingly current piece of calculation. To discover this involves incorporating this ‘new’ element in the genre of ‘insertion’. The students’ genre awareness and their genre system may grow. Next time, if remembered, such an element will occur as known (being thematic, not rhematic) belonging potentially to this particular mathematical genre.

There is in other words an intimate and direct relationship between the dynamics of themes and rhemes, and learning as combination of given/already acquired knowledge. But there is even a connection between
skills and the progression of new learning elements catered for by the curriculum, the textbooks and the teacher. The above case even represented a break with the didaktic/traditional educational expectation that any ‘major’ new element would normally be presented by the teacher first (which in the next step often leads to a mechanistic perception of ‘calculation’). As soon as there are ‘difficulties’, students may hesitate in trying to solve them.

If we look at this episode from the perspective of communicational triads, we can easily find a tight connection between the cognitive aspect (not understanding the possibility of negative quantities in this genre), the emotional aspect (frustration) and the act (giving up). The task had a form/structure, which implied a certain content and a certain addressivity. The answer is expected to have structural clarity, to be logically ‘true’ and to be performed so that it can be evaluated.

In the following I will keep one eye on the shift between given and new and the other on expressivity. In other words these two chosen, combined foci are my personal interrelated positionings of the texts in question. I could have focused any aspect of the presented, overall framework, but by giving priority to expressivity, this aspect becomes an object, a figure, and the other aspects a (back-)ground, both a result of my chosen position.

7. Positioning theme-rheme and expressivity

For a newborn child the utterance $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$ does not consist of theme and rheme. Theme has to be given and rheme partly recognizable to be meaningful as a whole. This particular utterance may for learners of mathematics function as (partly) meaningful in relation to how communication, language, and mathematical knowledge are built up, stage by stage for potential interpreters. I will give a quite extensive example of how a particular textbook in mathematics education for student teachers (Nygaard et al., 1999) explicitly has planned a whole chapter, in order, at the end of the day, to reach the above utterance. The textbook actually classifies itself in the subtitle as Mathematics and the didaktik of mathematics (my translation).

Functions and equations

In this chapter you will learn about the concept of function, polynomial functions and rational functions. In addition you will get an insight into the concepts ‘derivation’ and ‘integration’.

You will learn to solve equations and equation systems. We will also show how we can solve an equation by approximation using spreadsheets. In addition you will gain insight in extracts from the history of equation.
If you know the syllabi of functions from upper secondary school well, you may think that this chapter is strange. It is quite different from what you might expect, we are quite sure about that. However, why learn once again what one already knows?

If you do not know much about functions, it is rather doubtful whether it is a good idea to try the same method as the last time you failed to get the point. We therefore want to lay hold of other aspects of the theory of function, and focus more on the concepts themselves and study some different types of functions so that you can see what a function is, and learn to appreciate what a good idea functions represent. (Nygaard et al., 1999, p. 375, my translation)

Thus one of the sub-chapters concerns the history of equations, starting with 1st-degree equations. In a balanced mixture of mathematical genres such as reasoning, explanations, examples, tasks, and storytelling, we are brought narratively through the history of equations. We meet, in structured order, the history of the 1st-degree equation, the 1st-degree function, the history of the 2nd-degree equation, the 2nd-degree function, the history of the 3rd- and 4th-degree equation and the history of the 5th-degree equation. We end with the impossibility of solving 5th-degree equations with an algebraic formula for solution. The long and persistent narrative line represents a conscious will to combine new and given (mathematical themes and rhemes). Each new rheme part is accumulated and forms a building block for the next topic. In a historical perspective these new phenomena evolved into new mathematical genres of the time and they were further developed by adding a rheme part, which changed or expanded the former level of understanding and complexity. Through the narrative the textbook writers make (the history of) mathematics into mathematics education (didaktik) by positioning the development by certain discursive, didaktic grips.

Part of the story though contains some short glimpses of expressivity. The ‘race’ between Italian theorists to be the first to solve 3rd-degree equations (and to keep the secret) is also described. (Nygaard et al., 1999, pp. 410–411.) When moving from 3rd to 4th-degree equations, the following lines occur [the (…) indicates text I have left out]:

You learned to solve 2nd-degree equations in upper secondary school, now it probably will be fun [bil det nok gøy] to learn to solve the 3rd-degree equation. The 4th-degree equation we are afraid you have to wait to learn [må du dessverre vente med å lære] until you choose a more advanced course in mathematics. (Nygaard et al., 1999, p. 411) (…)

The history of the 5th-degree equation

We have now seen that during the 16th century formulas were found for solving the general third and 4th-degree equation. After this achievement it was natural
for mathematicians to start the work finding a formula for the general 5th-degree equation:

$$ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$$

This was done, and in this history the Norwegian mathematician Niels Henrik Abel (1802–1829) played one of the leading parts.

Abel first believed that he had found a formula for solving the 5th-degree equation, but he soon discovered that it was incorrect. (…) [In the margin there is a portrait of Abel accompanied by the text: “Niels Henrik Abel (1802–1829), perhaps the smartest Norwegian ever.”] What Abel was able to prove in 1824, was that there does not exist a formula for solving the general 5th-degree equation. How could Abel know that there did not exist an extremely complicated formula that he had not tried? (…) What Abel did was to find which property such a formula had to have, and then prove that irrespective of how you build up the solving formula, it will not in any case have the requested quality. Smart, don’t you agree?

Further it becomes clear that for \( n > 4 \), there is no general algebraic formula for solving an equation of the \( n \)th degree. Hence with this result from the 19th century we can just forget to try to get famous for finding an algebraic formula of solution for instance the general equation of eighth degree. (Nygaard et al., 1999, p. 413)

If we consider this text from the perspective of theme-rheme (or given and new), we should keep in mind that we are confronted with a simultaneous blurring of genres. There is a story, and stories have their own expectations or norms regarding the balance of theme and rheme, which is also culturally dependent. Moreover, there is a parallel, thematic reasoning that tries to explain. A main point here is that most student teachers may have experienced a rather mechanistic enculturation to algebra and their genre expectation to the field of functions is perhaps that tasks appear as something solvable, there is always an ‘answer’. In this text something suddenly occurs as not solvable. A major ideological ‘sense’ in doing school mathematics may abruptly disappear.

However, things turn out not to be simple. In more reduced understandings of the matter it may be said that equations of fifth degree “do not have a solution” or that equations of a higher degree than four are ‘impossible’. The theme strategy for these textbook authors is therefore directed, not only towards an evolving, step by step understanding of the ‘elegance’ of Abel’s proof, but even functions as a stepping-stone for the next topic, functions of \( n \)th degree (Nygaard et al., 1999, p. 413). Hence, the more overarching theme-rheme strategy in the entire chapter is to build up a careful succession of mathematical knowledge by a mixture of verbal and arithmetic utterances where the rhythm of the theme-rheme balances reflects the estimated increase of the learning capacity of the learners/readers, their openness to mathematical semiosis.
A didaktic point is that the diachronically told history may help some student teachers to understand that they are not ‘stupid’ if they do not grasp this total accumulated complexity immediately. It has taken 2000 years of human intellectual history to reach this understanding, a knowledge that normally is presented as (synchronously) given. The perspective can also make student teachers (and students) aware that the extensive genre knowledge their teachers and the researchers of mathematics at the cutting edge of theory in this field have acquired, has been built up over time, slowly and step by step, as theme-rheme alterations in the everyday of learning.

Theme-rheme balances are even important for positionings at a more general level. In fields such as chemistry and linguistics the theme part is expected as a necessary point of departure. Moreover, in any successful research a crucial part, the outcome, is expected to be new to deserve the notion of research. Further, theme and rheme are neither objective aspects nor the same for utterers and receivers. Nevertheless, the more communicators share the elements and the clearer the elements are presented, the more likely it is that one may find a general agreeable balance of what is theme and rheme (and hence what is the meaning or a possible intention). Imbalances of the teacher’s and the students’ conceptions of what is new and given is therefore a core point and the very dynamics of teaching and learning in education.

8. POSITIONING EMOTIONALITY AS IDEOLOGICAL

(...) the things we take for granted without inquiry or reflection are just the things which determine our conscious thinking and decide our conclusions. And these habits which lie below the level of reflection are just those which have been formed in the constant give and take of relationship with others. (Dewey, 1916, p. 22)

The above systemic framework is not categorical, but communicational and hence a relational invitation to concrete approaches. Thus for operationalisational purposes one needs a relational concept to combine the three main aspects as well as the dialogical dynamics of the micro (the utterance/text) and the macro (the genre/context) levels. Positioning is coined for this purpose. However positioning without any topical focus would have been just a purely abstract concept, and thus in itself empty. A researched object or a phenomenon therefore has to be semiotically focused to be specific, which will, according to the logic of the framework, create a figure and a ground, and accordingly an implicit discursive platform or position relative to the focus, within the framework. In the following example ‘emotionality’ in mathematics education is used to exemplify the focus for the positioning.
I underlined in the textbook extracts above places where ‘unexpected’ expressivity is breaking through in the text. I have also marked in the text, three formulations: perhaps the smartest Norwegian ever.” (...) Smart, don’t you agree? (...) we can just forget to try to get famous. The reason why I interpret the utterances as emotional, carrying more than ‘normal’ expressivity, is that, according to my traditional understanding of norms for mathematical writing, I do not expect mathematical texts to be particularly expressive. Methodologically and in Bühler’s terms I have read the text as a symptom. Or in Bakhtin’s term, I have positioned the text as expressivity. This particular expressivity can be interpreted as an emotional symptom of the authors’ engagement. However the marked text-elements can also be refuted as just referential symbols/words, with secondary importance for the ‘main’ intention of the text as a whole, namely understanding. Or they can even be interpreted as Bühlerian/Bakhtinian forms that signal or address students (of our time?) in a particular way, and should perhaps rather be seen as addressivity.

There are of course many ways of reading any text. In this particular case, I am also interested in ideologies of the utterances and the genres, in particular since this concerns education, and since a general positioning, or point of departure, in my own research has been that school and education are sites for particular ideological constraints (Braathe and Ongstad, 2001). Bakhtin holds that any utterance simply has to contain (tacit) values. The reason is that a genre help to preposition communicatively what we may expect and therefore is a functional, effective, and even a necessary grip in communication. Tacit implications and values of genres then will function as ideologies hidden in utterances (Bakhtin, 1986). Similarly Ricoeur (1981) explains that ideologies are something we think from, rather than on.

In school there is often a potential conflict between subject teachers’ engagement and students’ more instrumental attitudes. One particular ideology, that rarely is admitted, especially in mathematics education, is the hidden and sometimes open admiration of smartness and intelligence that may occur in the field (Braathe and Ongstad, 2001). Teachers of mathematics are probably often ‘winners’ in the race for good marks and high ranking. Besides mathematics is the school subject par excellence to be used for comparative purposes on a personal and even an international level. In this particular text an ideology seems to be that working with mathematics, at the end of the day, can be related to smartness. The textual strategic positioning of the three expressive elements may underpin such a reading/positioning of the text. The problematic conflict between egalitarian values and this tendency to admiration of intelligence is problematised more in detail in Braathe and Ongstad (2001).
9. VALIDATING WITHIN A TRIADIC FRAMEWORK

Rather than to conclude, using deductive or inductive strategies, I will end the presentation and exemplification of the framework with a problematization of what it may mean to validate relative to a systemic, open, triadic, semiotic, communicational framework. In Ongstad (2004a) I have presented in more detail some challenges connected to a paradigmatic move from a dyadic to a triadic view (or from language to communication). Here I will mostly raise self-critical questions, and answer them partly in an admitting way.

Counter-reading, triangulation of different readings and self-critique of the researcher’s (my) own positioning should also be conducted: is for instance the textbook writers’ way of writing dominant in their book of 719 pages? No, there is just a slight tendency. Can the particular discourse strategy of the authors be understood differently? Yes, it can be seen as a conscious grip to make their textbook less ‘serious’ in order to engage more students. Can a researcher’s particular interest in the affective domain lead to a twisted understanding of its epistemological value? Yes. In other words – semiotics and communication as approaches are multifunctional, and starting from triadic thinking does not necessarily create neat closures and categories, but may increase the openness of utterances. Still, this kind of openness seems closer not only to the concrete, daily reality of practicing teachers, but even to an increased complexity brought to the forum by ‘Wissenschaftstheorie’ (a general theory of science) the last decades.

Hence studies from (and of) different positionings will not necessarily give easy results and ‘correct’ answers. Analyses of positioning related to a triadic communicational framework should help tracing new, hidden aspects or ‘ideologies’, things we blindly take for given. And since the approach is systemic, it allows for an explicit considering of its own positioning(s) of validities, and can thus be related critically to over-all ideologies and positionings, such as ‘subjectivity’, ‘objectivity’ and ‘normativity’ (Habermas, 1984). Further, the approach is socio-semiotic in the sense that it considers ‘findings’ as systemically dependent of context/discourse. These contexts are not chaotic, but systemic as they tend to take the character of discourse or genre.

Finally, the triadic basis allows for flexibility and for the multifunctional. I would like to emphasize that analysis of positioning(s) in relation to utterances and genres does not completely refute an essentialist approach. My view differs partly from a post-structural standpoint. Thus there is room in the framework for Popper’s idea of a ‘third world’ (compare Ernest, 1998), as well as for Popper’s claim that the researchable should be anchored in a statement (a proposition) that can be tested (have the
ability to evaluated as false). The difference between a more essentialist approach and an open triadic one is that the latter approach admits and explains, firstly that communicational meaning is reduced to an object in the very moment the referential aspects are made into categories, and secondly that contexts are always dynamic and will be ‘killed’ once referential aspects are separated from a fuller communicational meaning.

By objectifying referential aspects of communication we may run into trouble in explaining how and why context is neutralized. This is especially the case once one moves from ‘nature’ to ‘culture’ as researched ‘objects’ or from the field of mathematics to the field of mathematics education or from linguistics to semiotics. Whilst mathematics and science as academic disciplines still may prefer to stick to traditional epistemological validities only, mathematical education or ‘Mathematik-didaktik’ cannot. Mathematics education is cultural, that is, a semiotic based field even if mathematics as a dynamic part of this field would be seen as ‘a hard science’. Didaktik as an approach has to take into consideration, simultaneously, the subjectivity of the learner, the ‘objectivity’ of the referential content, and the normativity/ intersubjectivity of the acts and the activity, not as separate categories, but as inseparable aspects. Hence they are didactically and hence educationally valid only in their blurred state. The necessary paradox though is that we have to and should focus on the parts as well. This is again the inevitable situation for practicing teachers. It is in the intertwined form that learners, teachers and researchers will have to relate to utterances and their communicational environments and contexts, the genres.

When we utter, we evaluate simultaneously the aesthetic, the epistemological and ethical aspects of communication, all at once and blurred. Form, structure, syntax and style are evaluated as aesthetics between a range from nice to ugly or the like. Similarly, references, content, and knowledge are surveyed by an epistemological logical true-false regime. Finally utterances as acts are considered ethically by evaluations ranging from good-bad (or fair-unfair). Hence validation is a question of balancing, meta-cognitively, three inseparable kinds of communication, the expressive, the referential and the addressive (Ongstad, 1997). Besides, depending of which main aspect will be the point of departure, one will, according to Habermas, have to relate respectively, simultaneously and triadically to truthfulness/veracity, truth, and fairness/usefulness. It is of course the context which at the end of the day can guide us to see what may be relevant. However, since context is systemic through the inevitable impact of genre and genre expectations, we will normally have to rely on and be guided by our genre knowledge. Let us try to imagine three different discursive contexts to each of the following three
utterances:

I. Student: I hate mathematics!
II. Student: The total sum for all whole numbers from 1 to 100 is 5050.
III. Student: I will sum up all numbers from 1 to 100 by using the calculator.

In the first assumed context (I) we may understand the utterance as strong expressivity and interpret it as an emotional statement. We have to consider whether the student actually means it, how truthful the utterance is. In context II we might tend to ask: “Is it true?” and in III we might consider the functional value of using the calculator. However all three main validities/aspects/dimensions are present in each of the three utterances. What we do by giving priority to one, seemingly dominant aspect, is to position the utterance assuming a given context. By positioning we firstly take a position, a point of view, a perspective, secondly focus, creating a fixated object, and finally take for granted an un-uttered context that may help giving the utterance a fuller meaning. Thus in Sáenz-Ludlow’s example about the difference (my example B further above) both the teacher and the student chose a specific position, they both fixated on an object (difference) and they both took for granted a shared context/genre in which the focused word could belong.

Regarding context II there exists a famous anecdote about Gauss, who as a 10-year old got as ‘punishment’ from his teacher the task to sum up all whole figures from 1 to 100. Gauss however solved this fast and intuitively by imagining that 100 + 1, 99 + 2, etc would give 50 times 101 = 5050. This story was used as an introduction to arithmetic sequences as a mathematical genre in upper secondary education in Norway in the 1950s. In addition to support the understanding the very logic of arithmetic sequences, to hear the story could have different ideological outcomes for the students. As I have touched upon, mathematics could be seen as elegant or beautiful. Or mathematics was simply functional, a rational tool, a system for acting. In other words, there are hidden ideologies blurred into the everyday of mathematics teaching and learning, and these tacit expectations are again connected to different mathematical/pedagogical genres.

In this article, I have outlined a triadic, semiotic, communicational framework. This framework basically combines two principles, the simultaneity of three inevitable aspects, all three necessary to establish communication, and the dialogical dynamics of concrete utterances and immanent, systemic resources. The framework is not an approach, but rather a map. It therefore needs operationalisation. One possibility is positioning(s) which is a relational concept. Therefore it needs to be combined with a chosen focus. I have aimed to illustrate some of the main characteristics and
potentials of the outlined framework, but not in any sense all. The overall idea has been to advocate for a broad socio-semiotic rather than a linguistic perspective, and hence for seeing both mathematics and mathematics education as triadic communication and not (only) as dyadic, verbal language. Such a shift makes things more complex, but by the same token even more relevant and adequate, and above all it demands and offers necessary new principles for discursive validation, a solution not yet found in dyadic approaches.

ACKNOWLEDGMENTS

I would like to thank Norma Presmeg, the anonymous reviewers, and especially Adalira Sáenz-Ludlow for helpful comments on an earlier draft of the article. Besides, the Norwegian Research Council has, through the financing of the project “Wholeness in teacher education?”, made this work possible.

NOTES

1. In Ernest (this volume) there is a recognition of the Hallidayan approach. However, there is still a principle and practical separation of the sign and the use of sign. Ernest’s positioning therefore still seems more Peircean (cf. the positive evaluation of Bruner’s early combination of Piaget and Peirce) than Hallidayan, in spite the fact that Ernest opens for bridging the gap between la langue and la parole, an implicit important step also in Halliday’s work. Ernest’s approach is still not systemic functional or fully pragmatic or triadic, even if it is communicational. By saying this I still find Ernest approach important and productive since it in principle breaks with the ideology of the static sign and can help understand the diachronic evolvement of mathematics education as a field (see last part of my article).

2. ‘Didaktic’ and ‘didaktik are written with k to illustrate that the concept is semantically related to the educational field of didaktik. For a fuller explanation, see Ongstad (1999a, b). The notion of ‘didactics’ is actually in use among some educationalist, but I have chosen to use ‘didaktik/didaktic’ to hint to the reference this concept has in most of Europe and in Latin America and as semantically different from the pejorative meaning ‘didactic’ may have among most native speakers of English. Last, but not least, didaktik is a major educational discipline in many countries, both in educational studies and in the studies of the school subjects.

3. The concept of positioning stems from many sources, such as military and economical strategy, electronic surveillance of the earth by satellites, discursive communicational theory, and study of student strategies in classrooms (Davies and Harré, 1990; Evans and Tsatsaroni, 1994). Langenhove and Harré (1993) even speak about theory of positioning. See also Harré and van Langenhove (1991). My own use of the concept is more related to Bakhtin’s notions of utterance and genre, and is supposed to be applied in a broad semiotic context (Ongstad, 2002a, b).
4. Brandist (2002) holds that Bühler’s ideas were known by key persons in the Bakhtin Circle in the Soviet Union. Valentin Voloshinov, for instance, seems to have translated parts of Bühler’s early work into Russian as early as in the 1920s.

5. Martin (1997) configurates the relationship between the two levels and between triadic aspects in the following way.

![Diagram](image)

Figure 4. Functional diversification of language (the small circle consisting of the aspects textual, interpersonal and ideational) and social context (the big circle consisting of mode, tenor and field, respectively) according to Martin (1997, p. 5). Thus for Hallidayians (or Systemic Functional Linguistics) language has three interrelated metafunctions: ideational, interpersonal and textual, and their function and relationship should be understood as follows: **Ideational resources are concerned with representation, interpersonal resources with interaction and textual resources with information flow. In SFL this intrinsic functional organization is projected on to context, redounding with the variables of field, tenor and mode – where field focuses on institutional practices, tenor on social relations and mode on channel.** (Martin, 1997, p. 4.)

6. The ‘correct’ answer is ‘of course’ 0, since divisions are supposed to be performed horizontally, using the imagined fraction line as a division mark. This is ‘proved’ by the mathematical fact that the rest this time is not E, as the left half part of 8, but 0, the left part when divided horizontally. While such a ‘play’ with forms may seem somewhat naive for mathematicians, the semiotics may be a contextual reality for novice learners.

7. For a problematisation of some ideologies in mathematics education in Norway, see Braathe and Ongstad (2001).


REFERENCES


Ernest, P.: This volume, A semiotic perspective of mathematical activity: the case of number.
Khisty, C.J.: This volume, *The significance of dialogue in problem-solving using Martin Buber’s triad – “I-thou-we”.*


Morgan, C.: This volume, *What Does Social Semiotics Have to Offer Mathematics Education Research?*


Ongstad, S.: 1999b, ‘Self-positioning(s) and students’ task reflexivity – a semiotic macro concept exemplified’, *Journal of Structural Learning & Intelligent Systems* 14(2), 125–152.


Saenz-Ludlow, A.: This volume, Classroom interpreting games as mediators in the construction of meaning.


SIGMUND ONGSTAD
Faculty of Education
Oslo University College
E-mail: Sigmund.Ongstad@lu.hio.no